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# Universal three-body parameter in ultracold $^4\text{He}^*$

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We have analyzed our recently measured three-body loss rate coefficient for a Bose-Einstein condensate of spin-polarized metastable triplet  $^4\text{He}$  atoms in terms of Efimov physics. The large value of the scattering length for these atoms, which provides access to the Efimov regime, arises from a nearby potential resonance. We find the loss coefficient to be consistent with the three-body parameter (3BP) found in alkali-metal experiments, where Feshbach resonances are used to tune the interaction. This provides evidence for a universal 3BP outside the group of alkali-metal elements. In addition, we give examples of other atomic systems without Feshbach resonances but with a large scattering length that would be interesting to analyze once precise measurements of three-body loss are available.

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## I. INTRODUCTION

When the short-range interaction between particles gives rise to (near-)resonant scattering, few-body properties are expected to become universal, i.e., irrespective of the precise nature of the interaction and therefore applicable to nucleons, atoms, or molecules [1]. Within universal few-body physics a hallmark prediction is the Efimov effect, in which three particles that interact via a resonant short-range attractive interaction exhibit an infinite series of three-body bound states, even in the regime where the two-body interaction does not support a bound state [2]. The first experimental evidence of Efimov trimers came from an ultracold trapped gas of atoms [3] by tuning the strength of the interaction via a Feshbach resonance [4]. In the context of ultracold atoms, the universal regime is realized when the  $s$ -wave scattering length  $a$ , characterizing the two-body interaction in the zero-energy limit, is much larger than the characteristic range of the interaction potential. Signatures of Efimov states are imprinted on trap loss caused by three-body recombination, which typically determines the lifetime of an ultracold trapped atomic gas or Bose-Einstein condensate. So far, observations of Efimov features have been made in ultracold quantum gases of bosons:  $^7\text{Li}$  [5–7],  $^{39}\text{K}$  [8],  $^{85}\text{Rb}$  [9], Cs [3,10,11], a three-spin component mixture of fermionic  $^6\text{Li}$  [12–14], and the Bose-Bose mixture  $^4\text{K} + ^{87}\text{Rb}$  [15].

In addition to the scattering length, a three-body parameter (3BP) is needed to fully describe the spectrum of Efimov trimers. The 3BP accounts for all the short-range information that is not contained in the scattering length, including a true three-body interaction. It can be parametrized as the location of the first Efimov resonance,  $a_-$ , on the  $a < 0$  side of a Feshbach resonance. Initially, the 3BP was thought to be very sensitive to details of the short-range interaction and therefore different for each (atomic) system [16]. However, experiments around different Feshbach resonances and with different alkali-metal atoms found the ratio  $|a_-|/r_{\text{vdW}}$  in a narrow range between 8 and 10 [5,9,11], where  $r_{\text{vdW}} = \frac{1}{2}(mC_6/\hbar^2)^{1/4}$  is the range of the tail of the two-body potential (also called the van der Waals length), with  $m$  the atomic mass and  $C_6$  the long-range coefficient. There is a vivid theoretical

debate on the physical origin of this universal 3BP [17–22]. Most work points towards a three-body repulsive barrier that prevents the three atoms from probing the short-range interaction. An important question is how general the universal 3BP is. Experimental data outside the group of alkali-metal atoms could shed light on this issue.

In this paper we investigate the possibility of extracting the 3BP from our recently measured three-body loss rate coefficient in a Bose-Einstein condensate (BEC) of metastable triplet helium-4 (denoted as  $^4\text{He}^*$ ) [23]. We will show that its value is consistent with those measured in alkali-metal systems, providing further experimental evidence of a universal 3BP. We will also discuss other atomic systems that can be analyzed in a similar fashion. The common feature is that in the absence of a Feshbach resonance, these atomic systems already have a scattering length that is much larger than the range of the potential. The mechanism for this is an almost resonant interaction potential, i.e., a bound state is almost degenerate with the collision threshold. This potential resonance is a simple single-channel effect. In contrast, a Feshbach resonance is a multichannel effect, where the width of the resonance introduces another length scale [4], which may give rise to nonuniversal physics. Therefore, potential resonances are more directly related to the universal description connected to a large scattering length than Feshbach resonances.

## II. THREE-BODY LOSS IN ALKALI METALS

To relate our work to the alkali-metal experiments, we first summarize how the 3BP is extracted from three-body loss measurements around a Feshbach resonance [1,3]. In the limit of  $|a| \gg r_{\text{vdW}}$  the three-body loss rate coefficient  $L_3$  for identical bosons is given by

$$L_3 = 3C_{\pm}(a)\frac{\hbar a^4}{m}, \quad (1)$$

where  $C_{\pm}(a)$  are dimensionless prefactors that depend on  $a$ . Here we assume that three atoms are lost from the trap in the event of three-body recombination. The scattering length  $a$  is tuned by a magnetic field from  $a > 0$  to  $a < 0$  through

resonance. The prefactors are given by

$$C_+(a) = 67.1e^{-2\eta_+} \{ \cos^2[s_0 \ln(a/a_+)] + \sinh^2 \eta_+ \} + 16.8(1 - e^{-4\eta_+}) \quad (2)$$

and

$$C_-(a) = \frac{4590 \sinh(2\eta_-)}{\sin^2[s_0 \ln(a/a_-)] + \sinh^2 \eta_-}, \quad (3)$$

respectively. On top of a strong  $a^4$  scaling,  $L_3$  shows, as a function of  $a$ , a series of resonances for  $a < 0$  and minima for  $a > 0$ , and the locations of these Efimov features are determined by  $a_+$  and  $a_-$ . The parameters  $\eta_{\pm}$  are related to the decay of the trimers into atom-dimer pairs and provide a width to the Efimov features. Experimentally  $a_{\pm}$  and  $\eta_{\pm}$  are obtained by fitting Eqs. (2) and (3) to the measured  $L_3$  spectrum as a function of  $a$ . For identical bosons  $s_0 = 1.00624$ , such that  $C_{\pm}(a) = C_{\pm}(22.7a)$ , and therefore  $a_+$  and  $a_-$  are defined only within a factor  $22.7^n$ ,  $n$  being an integer. Universal theory requires a single 3BP and therefore the Efimov features for  $a > 0$  and  $a < 0$  are related, namely, via the relation  $a_+/|a_-| = 0.96(3)$  [1]. A nonuniversal 3BP would manifest itself as random scatter of  $|a_-|$  values in a range between 1 and 22.7 for different systems. However, the ratio  $|a_-|/r_{\text{vdW}}$  was found in a narrow range between 8 and 10 for experiments with different alkali-metal atoms [5,9,11,18], indicating a universal 3BP [24].

### III. ANALYSIS OF THREE-BODY LOSS IN $^4\text{He}^*$

Recently we have measured the three-body loss rate coefficient in a  $^4\text{He}^*$  BEC, prepared in the high-field-seeking  $m = -1$  Zeeman substate, and obtained the value  $L_3 = 6.5(0.4)_{\text{stat}}(0.6)_{\text{sys}} \times 10^{-27} \text{ cm}^6 \text{ s}^{-1}$  [23]. For spin-polarized  $\text{He}^*$  Penning ionization is strongly suppressed [25] and three-body loss dominates the lifetime of a  $^4\text{He}^*$  BEC. Scattering of spin-polarized  $\text{He}^*$  is given by the  $5\Sigma_g^+$  potential, for which high-accuracy *ab initio* electronic structure calculations are available [26]. For  $^4\text{He}^* + ^4\text{He}^*$  this potential supports 15 vibrational states. The highest excited vibrational state is weakly bound, which gives rise to a nearby potential resonance. Its binding energy is  $h \times 91.35(6) \text{ MHz}$ , measured by two-photon spectroscopy [27], from which a quintet scattering length of  $141.96(9)a_0$  ( $a_0 = 0.05292 \text{ nm}$ ) was deduced, consistent with the *ab initio* theoretical value of  $144(4)a_0$  [26]. It is indeed much larger than the range of the potential, as  $r_{\text{vdW}} = 35a_0$  [28], such that  $a/r_{\text{vdW}} = 4.1$ . The binding energy of this weakly bound two-body state corresponds to  $4.4 \text{ mK}$ , which is much larger than the trap depth of about  $10 \text{ } \mu\text{K}$  and therefore both the formed dimer and the free atom leave the trap after three-body recombination. There are no broad Feshbach resonances in  $^4\text{He}^*$  because of the absence of nuclear spin [29].

We now consider Eq. (2) to find the set of  $a_+$  and  $\eta_+$  values that explains our observed value of  $L_3$ . Following the current convention, we present the 3BP in the form  $|a_-|/r_{\text{vdW}}$  by using the universal relation  $a_+/|a_-| = 0.96$ . In the alkali-metal experiments typically  $\eta_+ \approx \eta_-$  and therefore in the following we will only use  $\eta$ . In Fig. 1 we show two sets of solutions of Eq. (2) that match our measured  $L_3$  value, namely,  $|a_-|/r_{\text{vdW}} = 2.3$  (dashed lines) and  $7.7$  (solid lines),

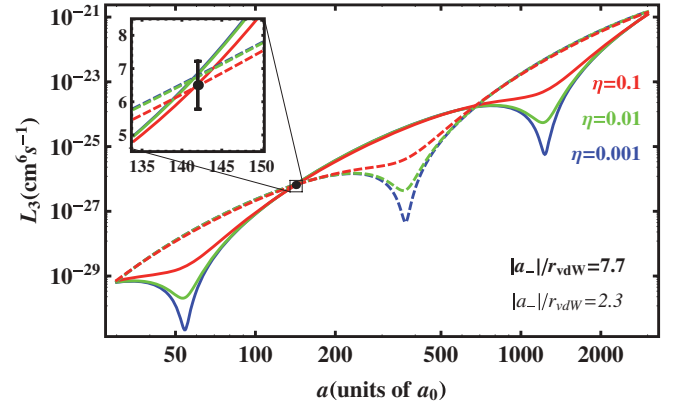


FIG. 1. (Color online) Universal three-body loss curves [Eq. (2)] for  $^4\text{He}^*$  with  $|a_-|/r_{\text{vdW}} = 2.3$  (dashed lines) and  $|a_-|/r_{\text{vdW}} = 7.7$  (solid lines), for different values of  $\eta$ , that match our measured  $L_3$  value (see inset).

for different values of  $\eta$ . In both cases our data point is located far outside an Efimov minimum, giving rise to a weak dependence of  $\eta$  on  $L_3$ . That is the reason why our  $L_3$  value, obtained for a single scattering length, provides information about  $a_-$ .

In Fig. 2 we show the set of solutions to Eq. (2) in  $(|a_-|/r_{\text{vdW}}, \eta)$  parameter space for our value of  $L_3$ , represented by the black solid line, with the gray shaded area reflecting the experimental uncertainty in our measured  $L_3$  value. Within the range of 1 to 22.7 for  $|a_-|/r_{\text{vdW}}$ , we indeed find two narrow regions of  $|a_-|/r_{\text{vdW}}$  around 2 and 8, provided that  $\eta$  is not too large. For  $\eta = 0.1$  we find  $|a_-|/r_{\text{vdW}} = 7.7(7)$  and  $2.3(2)$ . If  $\eta$  becomes larger than 0.5 the Efimov minima are washed out and their location becomes undefined, giving rise to a broad range of possible  $|a_-|/r_{\text{vdW}}$  values. For comparison, the 3BPs obtained from the different alkali-metal experiments are depicted by the colored symbols, with their numerical values

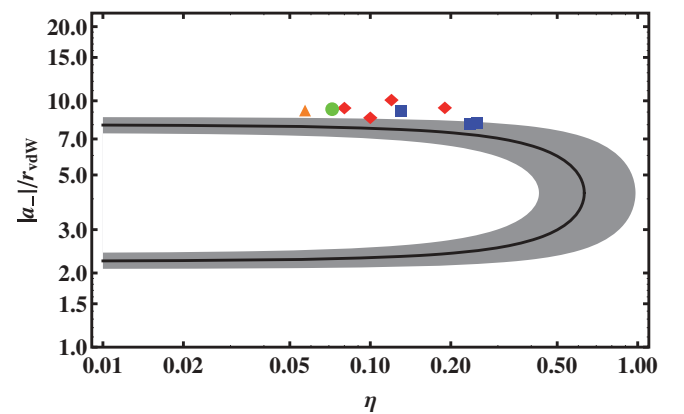


FIG. 2. (Color online) Graphic representation of the set of  $|a_-|/r_{\text{vdW}}$  and  $\eta$  values for which Eq. (2) matches our observed value of  $L_3$ , given by the black solid line, where the gray band corresponds to possible values based on our  $L_3$  error bar. Also indicated are the  $|a_-|/r_{\text{vdW}}$  values for the alkali-metal experiments: Cs, 8.6(2), 10.2(6), 9.5(8), 9.5(3) [11] (red diamonds),  $^7\text{Li}$  8.1(3) [5], 9.2(3) [6], 8.3(4) [7] (blue squares),  $^6\text{Li}$  9.3 [30] (green circle),  $^{85}\text{Rb}$  9.23(7) [9] (orange triangle), showing at the same time the observed  $\eta$  parameters.

given in the caption. We expect the value of  $\eta$  for  $^4\text{He}^*$  to be similar to those found in the alkali-metal systems, since Penning ionization will play no important role in the decay mechanism of the Efimov trimers. Figure 2 shows that our value is consistent with the 3BPs found in the alkali-metal system, considering the scatter shown in the available data and our uncertainty in  $L_3$ .

In our analysis we rely on two assumptions. The first assumption is that  $a/r_{\text{vdW}} = 4.1$  is sufficiently large for application of Eq. (2). Here we notice that the three-body loss data around a Feshbach resonance fit well for  $|a|$  larger than a few  $r_{\text{vdW}}$ . Effects beyond universal theory [31–33] may be present, but are small enough not to alter our conclusion. The second assumption is that three atoms are lost for each three-body recombination event. For  $a > 0$  additional resonances on top of the  $a^4$  scaling have been observed in three-body loss spectra [6,8,34]. Those features are explained by secondary atom-dimer collisions that are resonantly enhanced near  $a = a_*$ , where  $a_*$  is the atom-dimer Efimov resonance position [1], which effectively leads to an enhancement of the number of atoms lost in a three-body recombination event. The precise underlying mechanism, and therefore what to extract from these additional resonances, is still under debate [35–37]. Here we note that if we take  $|a_-|/r_{\text{vdW}} = 8$ , then  $a_* = 300a_0$ , which is far away from the actual value  $142a_0$ , such that secondary atom-dimer collisions are expected not to play a role for  $^4\text{He}^*$ .

#### IV. OTHER SYSTEMS

There are more atomic systems with a nearby potential resonance, for which a similar analysis as that performed for  $^4\text{He}^*$  can be done once a precise measurement of  $L_3$  becomes available. Alkali-metal atoms prepared in a spin-stretched state (i.e., electron and nuclear spin maximally aligned) scatter only in the triplet potential. Therefore alkali metals with a large triplet scattering length provide the opportunity to extract the 3BP obtained from three-body loss in the presence of a potential resonance. Two candidates are  $^{85}\text{Rb}$  [ $a_T = -388(3)a_0$  [38],  $r_{\text{vdW}} = 82a_0$ ] and  $\text{Cs}$  [ $a_T = 2440(24)a_0$  [39],  $r_{\text{vdW}} = 101a_0$ ]. An experimental challenge is to distinguish three-body loss from two-body loss processes, such as spin relaxation and hyperfine-changing collisions, especially in the case of  $\text{Cs}$  [40].

Another group of atoms that do not possess Feshbach resonances are the alkaline-earth-metal elements and  $\text{Yb}$ . In the electronic ground state the atoms have zero electron spin and therefore there is only a single two-body potential, which is of singlet character. Furthermore, the bosonic isotopes have zero nuclear spin and two-body loss processes are completely absent. An interesting example is  $\text{Ca}$ , for which potential

resonances show up for all the bosonic isotopes [41]. In the following we will discuss two isotopes of  $\text{Sr}$  and  $\text{Yb}$ , for which  $a$  is accurately known,  $a \gg r_{\text{vdW}}$ , and the first three-body loss measurements in BECs have already been reported.

For  $^{86}\text{Sr}$  [ $a = 798(12)a_0$  [42],  $r_{\text{vdW}} = 75a_0$ ], Stellmer *et al.* [43] report an upper limit of  $L_3 = 6(3) \times 10^{-24} \text{ cm}^6 \text{ s}^{-1}$ , which is one order of magnitude larger than maximally allowed by Eq. (2). The authors indicate that secondary collisions, possibly enhanced by a resonance in the atom-dimer cross section, may explain this discrepancy. We note that if one tentatively assumes that the scattering length is indeed near the atom-dimer resonance, i.e.,  $a_* \approx 800a_0$ , then  $a_- \approx -750a_0$  and thus  $|a_-|/r_{\text{vdW}} \approx 10$ . This is a hint that three-body loss in  $^{86}\text{Sr}$  is consistent with the universal 3BP.

For  $^{168}\text{Yb}$  [ $a = 252(3)a_0$  [44],  $r_{\text{vdW}} = 78a_0$ ], Sugawa *et al.* [45] report an upper limit of  $L_3 = 8.6(1.5) \times 10^{-28} \text{ cm}^6 \text{ s}^{-1}$ . If we perform a similar analysis as for  $^4\text{He}^*$  we find again two solutions of  $|a_-|/r_{\text{vdW}}$ . Taking the upper limit, one of the two solutions lies in a narrow range between 8 and 9. Here a smaller  $L_3$  leads to a larger  $|a_-|/r_{\text{vdW}}$ , and a value between 10 and 11 is reached when the reported  $L_3$  value is reduced by a factor of 2. This is a strong indication that three-body loss in  $^{168}\text{Yb}$  is also consistent with the universal 3BP.

#### V. CONCLUSIONS

We find our measured  $L_3$  coefficient in spin-polarized  $^4\text{He}^*$  to be consistent with the 3BP that was recently found in comparing measurements using alkali-metal atoms. We give further examples of atomic systems without a Feshbach resonance but in the presence of a nearby potential resonance for which the 3BP can be extracted from an accurately measured  $L_3$ , such as alkali-metal atoms in spin-stretched states and alkaline-earth-metal atoms. We find that the three-body loss measured in  $^{168}\text{Yb}$  strongly indicates consistency with the universal 3BP.

We provide experimental evidence for a universal 3BP, outside the alkali-metal group and in the absence of a Feshbach resonance. A universal 3BP means that short-range three-body physics is not relevant for the Efimov spectrum. This implies that not only three-body observables in the universal regime are fully determined by two-body physics, but four-body [46–48] and  $N$ -body ( $N > 4$ ) [49,50] observables as well.

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